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BOOSTER LOW-LEVEL RF FEEDBACK SYSTEMS

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The rf phase ϕ and the momentum p of a single particle executing phase oscillation are given by

$$\begin{cases} \dfrac{d\phi}{dt} = \omega_o - h\omega \\ \dfrac{dp}{dt} = \dfrac{ev_o}{2\pi R} \sin \phi \end{cases} \tag{1}$$

where

ω_o = angular frequency of rf

v_o = peak voltage of rf

h = harmonic number = 84

R = machine radius = 75.47 m

ω = $\omega(p)$ = particle revolution angular frequency.

For small oscillation we write

$$\phi = \phi_s + \psi, \qquad p = p_s + q$$

where the synchronous momentum p_s is determined by the guide magnetic field and the synchronous phase ϕ_s is determined by the peak rf voltage v_o through

$$\frac{dp_s}{dt} = \frac{ev_o}{2\pi R} \sin \phi_s .$$

Expanding Eq. (1) to linear terms in ψ and q we get

$$\begin{cases} \dfrac{d\psi}{dt} = -h\left(\dfrac{d\omega}{dp}\right)_s q + \left(\omega_o - h\omega_s - \dfrac{d\phi_s}{dt}\right) \equiv -Aq + \left(\omega_o - \bar{\omega}\right) \\ \\ \dfrac{dq}{dt} = \dfrac{ev_o \cos\phi_s}{2\pi R}\,\psi \equiv B\psi \end{cases} \tag{2}$$

where $\bar{\omega} \equiv h\omega_s + \dfrac{d\phi_s}{dt}$ and the synchronous particle revolution angular frequency ω_s is defined as $\omega_s \equiv \omega(p_s)$. We see that if the rf frequency ω_o is tuned to $\bar{\omega}$ then $\omega_o - \bar{\omega} = 0$ and Eq. (2) shows that the particle will simply oscillate about ϕ_s and p_s with angular frequency $\Omega = \sqrt{AB}$.

Since these equations are linear in ψ and q we can average them over all the particles in a beam bunch without altering their forms. Therefore, we can interpret ψ and q as those for the centroid of a bunch and consider Eq. (2) as giving the coherent phase oscillations of the beam bunches.

1. PHASE FEEDBACK (FAST)

We detect the rf phase ψ of the beam-bunch centroid and feed it back to the rf frequency ω_o. For this analysis we will write simply

$$\omega_o = \bar{\omega}_o + a\psi \qquad (a = \text{feedback constant})$$

and obtain from Eq. (2)

$$\begin{cases} \dfrac{d\psi}{dt} = -Aq + a\psi + \left(\bar{\omega}_o - \bar{\omega}\right) \\ \\ \dfrac{dq}{dt} = B\psi. \end{cases} \tag{3}$$

Remembering now that A, B, a, $\bar{\omega}_o$ and $\bar{\omega}$ are all slowly varying functions of time we can eliminate ψ and get

$$\frac{1}{B}\frac{d^2q}{dt^2} = -Aq + \frac{a}{B}\frac{dq}{dt} + \left(\overline{\omega}_o - \overline{\omega}\right)$$

or

$$\frac{d^2q}{dt^2} - a\frac{dq}{dt} + \Omega^2 q = \left(\overline{\omega}_o - \overline{\omega}\right)B. \qquad (\Omega^2 \equiv AB) \tag{4}$$

This is the basic equation and yields the following interpretations:

a. Coherent phase oscillations of the bunch caused by rf noise or other errors will be damped out by the phase feedback $\left(a\,\frac{dq}{dt}\text{ term}\right)$.

b. Although not necessary it is desirable that the coherent oscillation be critically damped, namely

$$\frac{d\omega_o}{d\psi} = a \simeq -2\Omega. \tag{5}$$

For 8 GeV operation the maximum phase oscillation frequency occurring about 2 msec after injection is about 30 kHz. At that time Eq. (5) gives

$$\frac{d\left(\frac{\omega_o}{2\pi}\right)}{d\psi} \simeq -60\text{ kHz/rad}. \tag{6}$$

Actually for most part of the acceleration cycle the phase oscillation frequency is considerably below 30 kHz. Hence a value for $d\left(\frac{\omega_o}{2\pi}\right)\Big/d\psi$ of -30 kHz/rad or even -20 kHz/rad is adequate.

c. After the coherent phase oscillation is damped out the equilibrium value of q is

$$q_e = \frac{\overline{\omega}_o - \overline{\omega}}{\Omega^2}B = \frac{\overline{\omega}_o - \overline{\omega}}{A} \tag{7}$$

with A and B as defined in Eq. (2).

d. The feedback <u>must</u> have a low frequency cut-off to let through the design slow time variations of the various coefficients in Eq. (3) or Eq. (4). The fundamental frequency of these time variations is, of course, 15 Hz. We should let through at least up to the tenth harmonic. The cut-off frequency should, therefore, be above 150 Hz.

e. Ideally, the phase feedback should be infinitely fast and perfectly linear. In practice, if the non-linearity is less than, say, 10% and the time delay is less than, say, $\frac{1}{10}$(phase oscillation period) ≃3 μsec the system should work all right.

2. RADIAL-POSITION FEEDBACK (SLOW)

The momentum deviation q_e is proportional to the radial position of the beam, the proportionality constant being given by the dispersion function x_p of the lattice through

$$x_e = x_p \frac{\Delta p}{p} = x_p \frac{q_e}{p} .$$

Therefore, we shall, in the following, sometime refer to q_e as the radial position.

To eliminate or reduce the residual or equilibrium radial position q_e given by Eq. (7) we feed q_e back into $\bar{\omega}_o$ and write

$$\bar{\omega}_o = \bar{\bar{\omega}}_o + bq_e \qquad \text{(b = feedback constant)}$$

where $\bar{\bar{\omega}}_o$ is, then, the rough rf frequency program given by the function generator. Substituting this in Eq. (7) we get

$$q_e = \frac{\overline{\overline{\omega}}_o - \overline{\omega}}{A-b} . \tag{8}$$

The radial-position error is, therefore, reduced by the factor $\frac{A}{A-b}$. As long as $|b|>>|A|$ the sign of b does not matter, in contrast to the phase-feedback constant. Evidently we do not care whether the beam is riding slightly outside or inside the central orbit. In any case the sign of $\overline{\overline{\omega}}_o - \overline{\omega}$ is unknown.

a. If, say, $|b| \simeq 10\ |A|$ we have

$$\begin{aligned} \left|\frac{d\overline{\omega}_o}{dx_e}\right| &= \frac{p}{x_p} \left|\frac{d\overline{\omega}_o}{dq_e}\right| = \frac{p}{x_p} |b| \simeq 10 \frac{p}{x_p} |A| \\ &= 10 \frac{h\omega}{x_p} \left|\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right| \end{aligned} \tag{9}$$

where we have inserted A as defined in Eq. (2) namely

$$A = h\frac{d\omega}{dp} = h\frac{\omega}{p}\left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)$$

with γ_t = transition - γ = 5.446. If the radial-position sensor is at the location of maximum dispersion (short straight section), x_p = 3.19 m. The maximum value of $|d\overline{\omega}_o/dx_e|$ occurs at injection when $\frac{h\omega}{2\pi} \simeq 30$ MHz, $\gamma = 1.213$,

$$\left|\frac{d\left(\frac{\overline{\omega}_o}{2\pi}\right)}{dx_e}\right| \simeq 600 \text{ kHz/cm} . \tag{10}$$

At 8 GeV, $\frac{h\omega}{2\pi} \simeq 50$ MHz, $\gamma = 9.526$, and

$$\left|\frac{d\left(\frac{\bar{\omega}_o}{2\pi}\right)}{dx_e}\right| \simeq 36 \text{ kHz/cm.} \tag{11}$$

b. In principle, there is no upper limit for $|b|$. Its value is limited only by the capability of the electronics: the larger the $|b|$, the larger the reduction of the radial-position error.

c. The radial-position feedback should have a high frequency cut-off below the lowest phase oscillation frequency. Otherwise, it will sense the instantaneous radial position q of the beam bunch instead of the equilibrium radial position q_e. In this case the radial-position feedback will also try to follow the coherent phase oscillation and results in a change of Ω^2. Putting

$$\bar{\omega}_o = \bar{\bar{\omega}}_o + bq$$

in Eq. (4) we get

$$\frac{d^2q}{dt^2} - a\frac{dq}{dt} + (\Omega^2 - bB)\, q = \left(\bar{\bar{\omega}}_o - \bar{\omega}\right)B.$$

The equilibrium radial position is, then

$$q_e = \frac{\left(\bar{\bar{\omega}}_o - \bar{\omega}\right)B}{\Omega^2 - bB} = \frac{\bar{\bar{\omega}}_o - \bar{\omega}}{A - b}$$

which is identical to Eq. (8) and gives the same reduction factor.

The disadvantage is twofold: First, since the phase oscillation frequency is changed from Ω to $\sim\sqrt{-bB}$ $\left(|b| >> |A|\right)$, $-bB$ <u>must</u> be positive and b must change sign at transition--

negative below transition and positive above transition. Secondly, the much larger phase oscillation frequency ($-bB \simeq 10\ \Omega^2$, say) demands larger (absolute) value of the phase feedback constant a if critical damping condition is to be retained.

A high frequency cut-off below ~400 Hz is about right.

d. Whether the rf frequency (ω_o) has a rough program ($\bar{\bar{\omega}}_o$) is not essential. A properly working radial-position feedback should be able to boot-strap an rf frequency program. In this case, however, the feedback constant $|b|$ will have to be much larger in order that the residual equilibrium radial position $|x_e|$ $\left(\text{or } |q_e|\right)$ needed for boot-strapping is reasonably small.

e. With a high frequency cut-off at ~400 Hz one has to be careful that the rough rf frequency program is sufficiently smooth. It is not important that the rough rf frequency program be right, it is only essential that this rough program not be so kinky that it is overly rich in frequency components above the cut-off.

3. SCENARIO

a. If the rough rf frequency program is perfect, namely $\bar{\bar{\omega}}_o = \bar{\omega}$, and the rf is noise-free, we do not need any feedback system, and can simply set a = b = 0.

b. The phase feedback only enforces internal consistency between the beam and the rf. It damps out the coherent phase oscillation so that the beam bunch will ride on the synchronous phase and the radial position as specified by the rf frequency. As such it does not know what the "right" frequency is supposed to be. Therefore, the phase feedback alone cannot correct for rf frequency "errors" and would allow the beam to stray out of the magnet radial aperture if the rf frequency as given by the rough program dictates it. However, together with a more-or-less "right" rough rf frequency program (so that the beam is always within the aperture) the phase feedback should greatly stabilize the beam against noise and errors.

c. The information of what the "right" frequency is supposed to be is contained only in the radial-position signal. For example, in a synchrocyclotron or an FFAG accelerator the radius (hence the synchronous momentum) of the beam is determined by the rf frequency and the beam is accelerated and moved to larger radius by modulating the rf frequency. In a synchrotron because the radius is fixed there is a "right" rf frequency program. Therefore, the radial-position feedback alone can correct for the imperfect (or even nonexistent) rough rf frequency program. Except without phase feedback any rf noise or other errors will lead to undamped coherent phase oscillations which will enlarge the phase-space area and eventually cause loss of beam.

d. A more detailed analysis should be made, taking into account the nonlinearity of phase oscillation and the transfer functions including roll-offs of the feedback electronics.